Invitational World Youth Mathematics Intercity

TEAM CONTEST

Time: 60 minutes

Instructions:
- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points. For Problems 1, 3, 5, 7 and 9, only answers are required. Partial credits will not be given. For Problems 2, 4, 6, 8 and 10, full solutions are required. Partial credits may be given.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator, calculating device, electronic devices or protractor are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

For Juries Use Only

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1. In the diagram below, replace each letter by a different number from 1, 2, 3, 4, 5, 6, 7, 8 and 9, so that the sum of the four numbers around each circle is equal to the number inside the circle.

Answer: 

```
 28  
A  B  C
  
D  E  F
  
G  H  I
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2. Find the sum of all four-digit perfect squares such that if each of its digits is reduced by the same amount, the resulting four-digit number is still a perfect square. (Different reduction amounts may be used for different perfect squares.)

Answer: __________________________
3. A hexagon has six angles of 120°. The lengths of four consecutive sides are 2000 cm, 2005 cm, 2010 cm and 2015 cm. Calculate the circumference, in cm, of this hexagon.

Answer: ____________________ cm
4. There is a 4×4 grid posted on the wall. Find the number of ways of placing two identical red counters and two identical blue counters on four different squares of the grid such that no column or row has two counters of the same color.

Answer: ________________ ways
What is the minimum size of a collection of perfect squares, with repetitions allowed, such that every positive integer up to 100 can be expressed as a sum of the numbers in the collection? A sum may consist of one number from the collection as well.

Answer: ___________________________ perfect squares
6. Kelly is younger than 100 and Kerry is older than 9. Kelly’s age becomes Kerry’s age when it is multiplied by a fraction whose denominator is 999 and whose numerator is a three-digit number with 5 as the tens digit. How many possible values of Kerry’s age are there?

Answer: ____________________________
7. In the diagram below, each hexagon contains one of the numbers 2, 4, 5, 6, 7, 18, 20 and 36. Each number appears once except that 6 appears twice. Each arrow contains one of the operations $-1$, $\div 2$, $+3$, $\times 3$, $+4$, $\times 4$, $\div 9$ and $+16$. Each operation appears once except that $-1$ appears twice. Complete the diagram so that each operation applied to the number in the preceding hexagon yields the number in the succeeding hexagon. Note that one of the hexagon succeeds no arrows while another one succeeds two arrows.

Answer:
8. $M$ is a point on the side $AB$ of a parallelogram $ABCD$ such that $AM : MB = 4 : 3$. $DM$ and $CM$ are perpendicular respectively to $AC$ and $BD$. If $BC = 5$ cm, find the area, in cm$^2$, of $ABCD$.

Answer: ____________ cm$^2$
9. Find the largest six-digit number with distinct digits which is a perfect square and its digits follow an increasing order from left to right.

Answer: ____________________
10. In a convex quadrangle $ABCD$, $\angle DAC = \angle DCA = 25^\circ$, $\angle BAC = 85^\circ$ and $\angle ACB = 30^\circ$. Find the measure, in degrees, of $\angle BDC$.

Answer: ______________