Invitational World Youth Mathematics Intercity Competition 2002

Team Contest

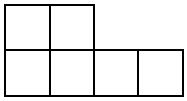
1. Let *m*, *n* and *p* be real numbers. If $a = x^{m+n} \cdot y^{p}$; $b = x^{n+p} \cdot y^{m}$; and $c = x^{p+m} \cdot y^{n}$, what is the numerical value of $a^{m-n} \cdot b^{n-p} \cdot c^{p-m}$?

2. Let $f(x) = \dfrac{bx+1}{2x+a}$ where *a* and *b* are constants such that $ab \neq 2$.

   (a) If $f(x) \cdot f(\dfrac{1}{x}) = k$ for all *x*, what is the numerical value of *k*?

   (b) Using the result of (a), if $f(x) \cdot f(\dfrac{1}{x}) = k$, then find the numerical value of *a* and *b*.

3. Prove or disprove that it is possible to form a rectangle using an odd number of copies of the figure shown in the diagram below.



4. Find all integers $x \geq y$, positive and negative, such that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{14}$

5. Four brothers divide 137 gold coins among themselves, no two receiving the same number. Each brother receives a number of gold coins equal to an integral multiple of that received by the next younger brother. How many gold coins does each brother receive? Find all solutions.

6. In $\triangle ABC$, $AB = BC$. A line through *B* cuts *AC* at *D* so that inradius of triangle *ABD* is equal to the exradius of triangle *CBD* opposite *B*. Prove that this common radius is equal to one quarter of the altitude from *C* to *AB*.

7. Two circles of radii *a* and *b* respectively touch each other externally. A third circle of radius *c* touches these two circles as well as one of their common tangents. Prove that $\dfrac{1}{\sqrt{c}} = \dfrac{1}{\sqrt{a}} + \dfrac{1}{\sqrt{b}}$.

8. Robbie the robot is locked in a solar panel and must get out through the hatch located at the centre of the panel, marked by $*$. Locked in with him are other dummy robots under his control. Each robot is mobile, but it can only move along a row or a column directly toward another robot, and can only stop when it bumps into the target robot, stopping in the empty space in front. In each scenario, four moves are allowed, where a continuous sequence of motions by the same robot counts as one move. Robbie is denoted by R.EXAMPLE

			A			
B			$*$			
R	C			D		

As an example, C-D-A-B and R-D-A is a two-move solution to the above scenario.

Scenario 1

A						B
			C			R
D						E

Scenario 2

			A			B
C			*		R	
			D			E

Scenario 3

A		B				C
			*			
R	D					E

Scenario 4

A		B				C
			*			
	R	D				E

Scenario 5

	A			B		C
D			*			
		R				E

Scenario 6

A				B		C
			*			
		R	D			E

Scenario 7

	A				B	
			*			C
	D		R			E

Scenario 8

A						B
C			*			
R	D					E